Lumped Parameter Building Model Calibration using Particle Swarm Optimization

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ABSTRACT
This paper presents a methodology for the automated calibration of deterministic lumped parameter models in building energy simulation using optimization methods. A heterogeneous model topology is proposed to represent a residential building archetype developed in the EnergyPlus simulation environment. The archetype model has previously been used to characterize the domestic building stock in Ireland. The automated calibration problem is solved as a least squares error problem solved using a local optimization method (Sequential Quadratic Programming) and two heuristics methods (Particle Swarm Optimization and Genetic Algorithm). It is shown that Particle Swarm Optimization provides the best performance for this particular problem and provides an inherent robustness under model uncertainty.

KEYWORDS
Lumped parameter building model, model calibration, particle swarm optimization

INTRODUCTION
The co-optimisation of the “all-island” Irish electricity grid and the demand-side Irish housing stock has the potential to provide researchers and utility planners with analytical guidance on electricity grid investment planning studies. Such study requires the development of building energy models in order to estimate the electrified domestic heat demand. Building Energy Model (BEM) archetypes have been successfully used in the scalable study of load flexibility in buildings (e.g., Neu et al. 2014). BEMS are often designed as iterative non-linear black-box solvers. On the other hand, deterministic electricity grid investment planning models (Wood and Wollenberg 1996) are expressed as Mixed Integer Linear Programs (MILP). Therefore, linear building models must be obtained. This article compares the performance of Sequential Quadratic Programming (SQP), Particle Swarm Optimization (PSO) and Genetic Algorithms (GA) in automated model calibration of lumped parameter building models using BEMS performance data. First, introductory information provides the scope of

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this paper. Then, a lumped parameter model is designed to capture the thermal performance of a detached house. The algorithms are evaluated when exact building geometry information is provided (nominal scenario) and when there exists a moderate building geometry information mismatch (uncertain scenario). The current paper closes with the conclusions of this work.

**BACKGROUND**

**Lumped Parameter Building Model Calibration**

Lumped parameter models are an alternative representation of heat transfer through building elements. This approach is based on the electrical analogy method (Robertson and Gross, 1958), in which electric resistances and capacitances represent the analogue of thermal resistances and capacitances of material layers. The resulting multi-nodal model is simplified by lumping parameters together, hence the name of the method. The heat balance at a thermal node $n$ is modelled as the first order differential equation:

$$C_n \frac{dT_n}{dt} = \sum_{i \in I} \frac{T_i - T_n}{R_i} + Q_n$$

(1)

where $R_i$ is the thermal resistance between elements $i$ and $n$, $C_n$ is the thermal capacitance of the node, $T_n$ represents the node temperature and $Q_n$ is the heat fluxes applied to the node. The set $I$ includes all nodes connected to node $n$. The automated calibration of lumped parameter building models can be posed as the error minimization between synthetic or metered building data and the model response (Coakley et al. 2014). Minimization routines based on gradient-descent optimization methods such as Sequential Quadratic Programming (SQP) require an accurate initial point (Gouda et al. 2002) or a multi-start heuristic technique (Pavlak et al. 2013).

Global heuristic optimization algorithms have been used in the past to calibrate lumped parameter building energy models. Wang and Xu (2006) employed Genetic Algorithms (GA) to calibrate a heterogeneous building envelope and internal mass using building data. Fraise et al. (2011) identified the parameters of a 4R5C model using Genetic Algorithms and constant step tests on a logarithmic time scale. The current article uses Particle Swarm Optimization, which is claimed to be more computationally efficient (less number of objective function evaluations) than Genetic Algorithms, although this property is known to be problem dependent (Hassan et al 2004).

**Particle Swarm Optimization**

Particle Swarm Optimization (PSO), is a population-based heuristic optimization algorithm which is inspired in the study of animal flocks (Kennedy and Eberhart 1995). In PSO, the performance of an individual member $i$ (denoted particle) evolves due to the influence of the population (denoted swarm), and vice-versa. Each particle is associated with a position $p(i,l)$ and a velocity $v(i,l)$, where the index $l$ denotes the current algorithm iteration. At each iteration the algorithm clusters particles in *neighbourhoods* (random subsets of the swarm) and updates the velocities and positions
of all particles based on their previous performance (i.e. its self-improvement) and their neighbourhood performance (i.e. its social improvement) via the weighted sum

\[ v(i, l + 1) = \omega v(i, l) + \alpha [u_1^T (p_{\text{best}}(i, l) - p(i, l))] + \beta [u_2^T (p_{\text{Nbest}}(i, l) - p(i, l))] \]  

(2)

where \( p_{\text{best}}(i, l) \) is the best performing solution known to the particle, \( p_{\text{Nbest}}(i, l) \) is the best particle in the neighbourhood, \( \omega \) is the inertia weight, \( \alpha \) and \( \beta \) are the self adjustment weight and the social adjustment weight and \( u_1 \) and \( u_2 \) are uniformly distributed random vectors of appropriate dimensions. Once the velocities are updated, the positions are then updated

\[ p(i, l + 1) = p(i, l) + v(i, l + 1) \]  

(3)

If a variable of particle \( p(i, l + 1) \) exceeds its bounds, then such element is set to its upper/lower bound. When a convergence criterion is reached, \( p_{\text{best}} \) becomes the optimal solution of the optimization algorithm. A more detailed explanation of the algorithm can be found in (Mathworks 2015).

**METHODOLOGY**

**Case study: Detached house archetype**

The work in this paper focuses on the simplified modelling and calibration of an EnergyPlus detached house archetype model representative of houses built in Ireland before 1985 (Neu et al. 2013). This archetype is modelled with double glazing windows \((u_{\text{win}} = 2.88 \ [W/m^2K], \ g_{\text{win}} = 0.759)\), heavy solid wall construction \((U_{\text{value}} = 2.27 \ [W/m^2K])\), a roof \((U_{\text{value}} = 6.301 \ [W/m^2K])\) and a moderate infiltration rate (0.67 ACH per zone, 2.1 ACH in the attic zone). Each of the 13 thermal zones is serviced with a direct electric resistance heater. The building also features an attic zone. Weather conditions correspond to the Dublin IWEC weather file (ASHRAE 2001). Internal gains and ventilation loads were omitted. The archetype model features temperature and wind dependent infiltration rate, adaptive heat transfer coefficient algorithms for all surfaces and an air mass capacitance multiplier of 11 included in order to emulate realistic building behaviour (Greensfelder et al. 2011).

Figure 1 shows the proposed heterogeneous lumped parameter building model topology associated with this archetype. The multi-zone building is approximated as a two-zone dwelling by using the average room temperature \((T_r)\) of the heated zones, defined as

\[ T_r = \frac{\sum_{i=1}^{n_r} T_i V_i}{\sum_{i=1}^{n_r} V_i} \]  

(4)

where \( T_i \) and \( V_i \) corresponds to the temperature and volume in the \( i \)th thermal zone, and \( n_r \) is the number of rooms. Node \( T_{\text{amb}} \) represents the dry-bulb outdoor temperature. Nodes \( C_{w1} \) and \( C_{w2} \) model the outer and inner leaves of the external walls. The solar gains due to solar radiation on the walls, \( Q_{s\text{wall}} \) are applied directly to node \( C_{w1} \). \( C_a \) represents the capacitance of the room air mass with room
temperature $T_r$. Node $C_{\text{int}}$ captures the thermal mass of the internal partitions and other slow dynamics. The window solar heat gain $Q_{\text{swin}}$ and the heating power input $Q_{\text{heat}}$ are split between $C_a$ and $C_{\text{int}}$ via parameters $f_1$ and $f_2$. Node $C_{\text{ceil}}$ models the ceiling between the room node $C_a$ and the attic node $C_{\text{attic}}$. The solar gains due to incident solar radiation on the roof surfaces, $Q_{s\text{roof}}$, are applied to the roof node $C_{\text{roof}}$. Finally, a ground node ($C_{\text{gnd}}$) is added to model the heat transfer between the conditioned volume and the foundations. The temperature $T_{\text{gnd}}$ models the temperature under the conditioned volume (boundary condition ‘Ground’ in EnergyPlus) and it is assumed to be constant at 18 °C (default value in EnergyPlus).

![Proposed lumped-parameter building model topology](image)

**Figure 1. Proposed lumped-parameter building model topology**

Two heating schedules were defined: one in the morning (7AM to 9AM) and another in the evening (5PM to 11PM). The temperature set-point is deemed to be 21°C for living areas and 18 °C for all other thermal zones. These set-points and schedules are representative of domestic heating requirements in Ireland (SEAI 2012). Since one of the rooms was deemed to be a living space, the effective set-point of the combined single zone, after room temperature weighting, is 18.3 °C.

**Implementation of the Cost Function**

In the context of automated lumped parameter building model calibration, the particle position corresponds to the candidate model parameters:

$$p(i) = [R_{\text{ext}}(i) \ldots R_{\text{ext}2}(i) \ldots C_{w1}(i) \ldots C_{\text{ceil}}(i)]$$

All resistance values were bounded between 0.001 and 1.0 $[m^2K/W]$, with the exception of the ceiling resistances, for which the upper limit was deemed to be 2.0 $[m^2K/W]$ because the baseline archetype was already insulated ($R=1.07$ $[m^2K/W]$).
All the capacitances were bounded between $1 \times 10^4$ and $1 \times 10^8$ $[\text{J/kgK}]$. The dimensionless factors $f_1$ and $f_2$ were bounded between 0 and 1. The cost function is evaluated using a MATLAB script which retrieves the model parameters from each particle, creates a continuous-time state space model based on the lumped parameter model equations (similar to Equation 2), discretizes the model and evaluates its performance. Geometry information is initially assumed to match the target building (archetype model) and therefore only the lumped parameters need to be calibrated. The cost function is defined as the root mean square error (RMSE)

$$J(p(i)) = \sum_{k=0}^{N_c} \frac{(T_{r,\text{data},k}-T_{r,k,l})^2}{N_c} + \sum_{k=0}^{N_c} \frac{(T_{\text{att,\text{data},k}}-T_{\text{att},k,l})^2}{N_c}$$

(6)

where $T_{r,\text{data},k}$ and $T_{\text{att,\text{data},k}}$ correspond to the weighted room and attic temperature time-series generated with EnergyPlus, $T_{r,k,l}$ and $T_{\text{att},k,l}$ correspond to the room and attic temperature response of the model and $N_c$ is the calibration horizon. Besides the RMSE, two additional building performance metrics were considered. The first is the Mean Absolute Error (MAE), which is the difference between the synthetic data and the model response (Mustafaraj et al. 2011). The second is the Mean Biased Error (MBE), which signals whether a model underestimates or overestimates model response (Mustafaraj et al. 2014).

This work uses the PSO and GA implementation provided with the MATLAB Global Optimization Toolbox (Mathworks 2015) and the SQP implementation provided with the MATLAB Optimization Toolbox (Mathworks 2015b). PSO and GA were run 20 times in order to ensure that an accurate solution is obtained, given the randomness in seed generation. The default tuning parameters were used, with the exception of the swarm size which is 200 (10 times the number of calibration variables) and the tolerance of the cost function (set to 0.001 [K]).

RESULTS
Nominal calibration performance
Since the interest of this study corresponds to heating mode only, synthetic data was generated for five winter months. The data set was split into three months of calibration data (mid-October to mid-January) and two months of validation data (mid-January to mid-March). Table 1 shows the best calibration performance of the SQP algorithm and the best calibration performance out of 20 runs of the PSO and GA algorithm.

Table 1. Performance metrics – full building geometry information

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>RMSE\textsubscript{calib} [K]</th>
<th>RMSE\textsubscript{valid} [K]</th>
<th>MAE\textsubscript{calib} [K]</th>
<th>MAE\textsubscript{valid} [K]</th>
<th>MBE\textsubscript{calib} [%]</th>
<th>MBE\textsubscript{valid} [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQP</td>
<td>0.385</td>
<td>0.538</td>
<td>0.308</td>
<td>0.442</td>
<td>0.153</td>
<td>0.788</td>
</tr>
<tr>
<td>PSO</td>
<td>0.296</td>
<td>0.442</td>
<td>0.230</td>
<td>0.342</td>
<td>-0.0465</td>
<td>0.253</td>
</tr>
<tr>
<td>GA</td>
<td>0.441</td>
<td>0.5495</td>
<td>0.348</td>
<td>0.454</td>
<td>0.226</td>
<td>0.614</td>
</tr>
</tbody>
</table>
PSO clearly outperforms the other two methods for this particular building, which is represented in lower performance metrics. Using the SQP algorithm results in an accurately calibrated model which marginally outperforms GA. The likely explanation of this occurrence is the accuracy of initial point provided to the algorithm, given that full geometry information and full material information is known a priori.

Figure 2 shows the model response of the calibrated models, between February 23 to February 28. All methods seem to calibrate the model response to a certain level of accuracy. Model response can vary due to thermal inertia and external weather conditions; hence a residuals comparison on the time-domain does not provide any meaningful information. Figure 3 allows to study the model residuals model performance via $MAE$ histograms. The vertical axis represents the frequency (in hours) for which the model results on a certain error level. While the SQP (left) and GA (right) algorithms result on a disperse error distribution (residuals between 0 and 1 K), PSO residuals (center) result on concentrated error distribution below 0.5 K, which shows that the model calibrated via the PSO algorithm is more accurate.

![Model response and residuals](image1)

**Figure 2. Model response and residuals (Feb 23 to Feb 28)**

![Residual Histograms](image2)

**Figure 3. Residual Histograms (Jan 15th to March 31st)**

**Calibration performance under uncertain building geometry information**

The same experiment was run with surrogated data generated with an archetype resized to 120% of its original dimensions. The algorithms were not provided with updated geometry information. Table 2 shows that the PSO algorithm was able to robustly adapt to this model variation, meaning that it provides metrics similar to a full information scenario (Table 1). The SQP and GA methods underperform with respect to the previous scenario, which was expected for the SQP algorithm as it is attracted to a local
minimum nearby the initial point. The GA algorithm did not present any adaptation capabilities for this particular problem and it is outperformed by the other methods. Note that this result is valid for the current case study only, and it has been obtained with the algorithm configuration stated in the methodology. Further work is required to study model calibration performance with other PSO and GA tuning parameters.

Table 2. Performance metrics volumetric variation of +20%

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>RMSE\textsubscript{calib} [K]</th>
<th>RMSE\textsubscript{valid} [K]</th>
<th>MAE\textsubscript{calib} [K]</th>
<th>MAE\textsubscript{valid} [K]</th>
<th>MBE\textsubscript{calib} [%]</th>
<th>MBE\textsubscript{valid} [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQP</td>
<td>0.630</td>
<td>0.661</td>
<td>0.509</td>
<td>0.530</td>
<td>0.309</td>
<td>-0.333</td>
</tr>
<tr>
<td>PSO</td>
<td>0.310</td>
<td>0.436</td>
<td>0.240</td>
<td>0.368</td>
<td>0.023</td>
<td>-0.579</td>
</tr>
<tr>
<td>GA</td>
<td>0.667</td>
<td>0.794</td>
<td>0.540</td>
<td>0.628</td>
<td>-0.045</td>
<td>-0.378</td>
</tr>
</tbody>
</table>

Computational Time

Table 3 shows the computational time of the best performing iteration and the mean of all calibration runs (in parenthesis). This work was implemented on a Dell Precision T1700 computer, with 8 cores and 8 GB of memory. The PSO and GA algorithms used 4 cores only. The GA algorithm exits earlier than PSO but with a suboptimal solution (Tables 1 and 2). A possible explanation is that the GA algorithms are based on the evolution is of its initial seed (‘survival of the fittest’) whereas PSO algorithms are able to perform a wider exploration of the search space thanks to ‘collective intelligence’, which results in better solutions at the expense of longer runs (Hasan, 2004).

Table 3. Computational Performance (in seconds)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SQP</th>
<th>PSO</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full knowledge</td>
<td>85 (89)</td>
<td>1124 (772)</td>
<td>358 (339)</td>
</tr>
<tr>
<td>Uncertain geometry</td>
<td>106 (107)</td>
<td>879 (689)</td>
<td>454 (356)</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper introduces the calibration of lumped parameter thermal networks using Particle Swarm Optimization. PSO can be easily used for the deterministic calibration of lumped parameter building models when there is insufficient information required to obtain an accurate initial point for gradient-descent optimization. Furthermore, PSO was able to calibrate the model under incomplete information. GA were found to be underperforming for this particular dwelling, likely exiting early with suboptimal solutions. Further work is required for a more rigorous comparison of all methods.

ACKNOWLEDGEMENTS

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REFERENCES